EFFECT OF BOUNDARY CONDITIONS ON THE HEAT TRANSFER LAW FOR A TURBULENT BOUNDARY LAYER

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The heat transfer at an impermeable plate has been experimentally investigated under various boundary conditions. The conservativeness of the heat transfer law $St_0 = f(Re_T^{**})$ is demonstrated for a monotonic increase of temperature and heat flux along the surface.

The question of the effect of boundary conditions on the development of a thermal turbulent boundary layer has attracted the attention of numerous investigators [1–5].

The method of calculating heat transfer and friction first proposed in [6, 7] has recently been widely adopted. This method is based on the idea that in integrating the boundary layer equations in the case of an arbitrary distribution of the wall-flow temperature difference, and also a longitudinal pressure gradient, on a certain interval of the bounding conditions it is possible to use the laws of friction and heat transfer obtained for an isothermal flow over a flat plate. It is used for calculating the turbulent layer on permeable surfaces and can be extended to other complex boundary conditions [8–10].

The energy equation for plane flow can be written in the form [10]

$$\frac{d\,\mathrm{Re}_T^{**}}{d\bar{x}} + \frac{\mathrm{Re}_T^{**}}{\Delta T}\frac{d\Delta T}{d\bar{x}} = \mathrm{St}\,\mathrm{Re}_L, \tag{1}$$

where

$$\mathrm{Re}_T^{**} = \frac{\rho_0 w_0 \delta_T^{**}}{\mu_0},\quad \Delta T = T_w - T_0;$$

$$\mathrm{Re}_L = \frac{L\rho_0 w_0}{\mu_0},\quad \bar{x} = \frac{x}{L},\quad \mathrm{St} = \frac{\alpha}{\gamma_0 w_0 c_{p_0}}.$$

For the zero-gradient flow of an incompressible fluid with constant physical parameters at constant wall temperature we have

$$\mathrm{St} = \frac{A}{\mathrm{Re}_T^{**m}\,\mathrm{Pr}^n}. \tag{2}$$

We assume that this heat transfer law is conservative with respect to longitudinal variations of heat flux and wall temperature. Then the solution of Eq. (1) for a given distribution $\Delta T = f(\bar{x})$ along the wall is the integral

$$\mathrm{Re}_T^{**} = \frac{1}{\Delta T}\left[\frac{(1+m)A}{\mathrm{Pr}^n}\mathrm{Re}_L \int \Delta T^{1+m} d\bar{x} + c\right]^{\frac{1}{1+m}} \tag{3}$$

and, correspondingly, for a given law of heat load distribution $q_w = f(\bar{x})$

$$\mathrm{Re}_T^{**} = \left[\frac{A}{\mathrm{Pr}^n}\,\mathrm{Re}_L\,\frac{1}{q_w}\left(\int q_w d\bar{x} + c\right)\right]^{\frac{1}{1+m}}, \tag{4}$$

where c is a constant of integration determined from the boundary conditions.

Knowing Re_T^{**} from (3) and (4), we can use Eq. (2) to calculate the heat transfer in a given section. In order to check the conservativeness of the heat transfer law with respect to changes in the boundary conditions, we performed the special experiments described below.

These experiments were conducted in an open-circuit wind tunnel with a maximum velocity of up to 50 m/sec in the rectangular (120 × 120) working section (Fig. 1).

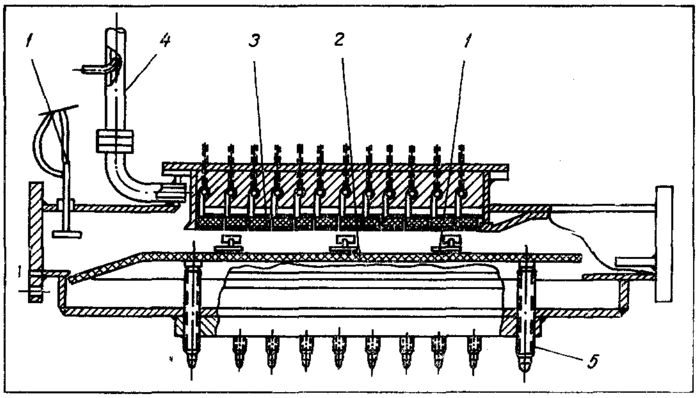

Fig. 1. Schematic view of working section: 1) Prandtl tube; 2) flexible strip; 3) calorimeters; 4) ejector for sucking off boundary layer; 5) flexible-strip regulating screws.

A convergent section (contraction 4:1) ensured a sufficiently uniform velocity profile at the inlet to the working section.

An electrically heated sectional plate-calorimeter 600 mm long was installed in the top of the working section, facing an elastic steel strip used to keep the velocity constant along the length of the channel. This was achieved by varying the channel cross section to compensate for the growth of the boundary layer.

The experimental plate consisted of 12 individual sections, each with a 0.1-mm stainless (1Cr18Ni9Ti) foil plate heater bonded to an asbestos-cement base. On top was a brass plate measuring 50 × 70 × 3 insulated from the heater by a thin sheet of mica. This plate had two deep notches parallel to the channel axis. Thus, the experimental plate was guarded, as it were, by protective heaters, which considerably reduced possible heat losses to the side walls of the channel. The inside face of the brass plate carried 0.2-mm thermocouples in glass-fiber insulation. To improve the thermal contact between the brass plate and the heater, tension was applied to the entire section by means of six 2-mm bolts. All 12 sections were assembled on a common frame with a sharp leading edge 30 mm long. The boundary layer that developed on the wall of the working section in front of this edge was sucked off by an ejector. The optimal suction was determined as follows. A 0.1-mm nichrome wire was introduced parallel to the edge and 10 mm away upstream. The wire was electrically heated, as a result of which a thin layer of heated air was formed downstream behind it; this was clearly visible through the windows in the sides of the channel in an IAB-451 shadow instrument.

At the optimum suction the wake was parallel to the heat transfer surface. At the same time, we measured the velocity field by means of a Prandtl tube in the section of the working chamber corresponding to the edge of the ejector orifice. These measurements showed that at the optimum suction the thickness of the boundary layer at the beginning of the orifice was practically equal to zero.

The heat losses of all the sections were determined in calibration experiments with the cavity of the working chamber filled with thermal insulation at T_w = const and in the presence of a temperature drop between neighboring sections. These losses did not exceed 10–15% of the total heat extracted from the plate.

The experimental data were correlated by the relation

$$St_0 = f(Pe_T^{**}),$$

$$St_0 = \frac{St}{\Psi}, \quad St = \frac{q_w}{\gamma_0 w_0 c_{p_0} (T_w - T_0)},$$

$$\Psi \simeq \left(\frac{T_w}{T_0}\right)^{-0.5}, \quad Pe_T^{**} = \frac{w_0 \delta_T^{**}}{a},$$

where

$$\delta_T^{**} = \frac{\int_0^x q_w dx}{\gamma_0 w_0 c_{p_0} (T_w - T_0)}. \tag{5}$$

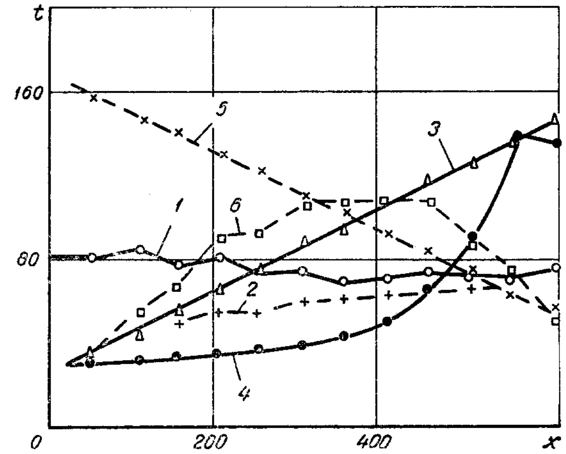

Fig. 2. Variation of wall temperature for various boundary conditions: 1) ΔT = const; 2) q_w = const; 3) $\Delta T = b + d_0\bar{x}$; 4) $q_w = q_0 \exp k\bar{x}$; 5) $\Delta T = b - d_0\bar{x}$; 6) $q_w = q_0 \sin \pi\bar{x}$; t in °C, x in mm.

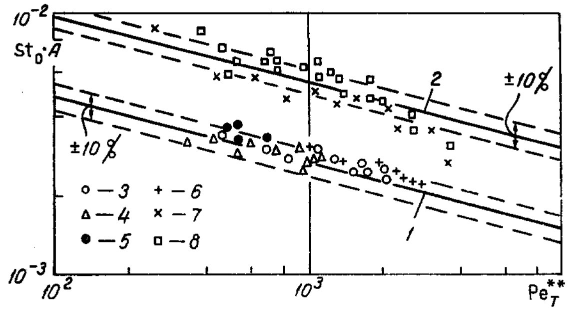

Fig. 3. Experimental data correlated as $St_0 = f(Pe_T^{**})$: 1) A = 1; 2) A = 2; 3) ΔT = const; 4) $\Delta T = b + d_0\bar{x}$; 5) $q_w = q_0 \exp k\bar{x}$; 6) q_w = const; 7) $\Delta T = b - d_0\bar{x}$; 8) $q_w = q_0 \sin \pi\bar{x}$.

Expression (5) follows directly from the energy equation (1). The first series of experiments was carried out with the boundary conditions: ΔT = const; q_w = const; $\Delta T = b + d_0\bar{x}$ $(d_0 > 0)$; $q_w = q_0 \exp k\bar{x}$, with the wall temperature and the heat flux increasing monotonically along the length of the plate (Fig. 2).

As may be seen from curve 1 of Fig. 3, irrespective of the form of the boundary conditions the experimental points are grouped around the curve

$$St_0 = \frac{0.0126}{Pe_T^{**0.25} Pr^{0.5}}. \tag{6}$$

Relation (6) was previously obtained in [10] on the basis of [11] for the condition ΔT = const.

In Fig. 4 the same experimental data are correlated in the form

$$St_0 = f(Re_x), \tag{7}$$

where

$$Re_x = \frac{\rho_0 w_0 x}{\mu_0},$$

and x is the length reckoned from the beginning of the turbulent boundary layer, which in our experiments almost coincided with the beginning of the plate. As

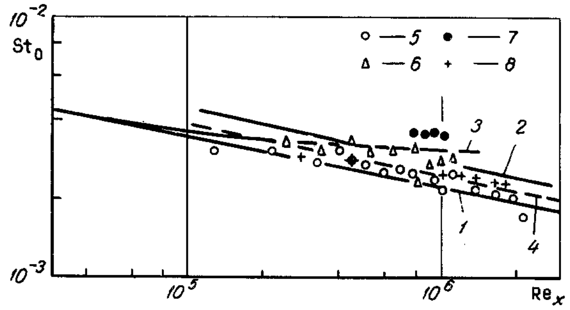

Fig. 4. Experimental data correlated as $St_0 = f(Re_x)$: 1) Eq. (8); 2) (10); 3) (14); 4) (12); 5) ΔT = const; 6) $\Delta T = b + d_0\bar{x}$; 7) $q_w = q_0 \exp k\bar{x}$; 8) q_w = const.

may be seen from Fig. 4, the experimental points are distinctly stratified according to the experimental regime. Using Eqs. (1) and (6) we can find relations (7) for each of these boundary conditions.

For ΔT = const we have

$$St_0 = \frac{0.0288}{Re_x^{0.2}\,Pr^{0.6}}. \tag{8}$$

For $\Delta T = b + d_0\bar{x}$, m = 0.25, n = 0.75

$$Re_T^{**} = \frac{0.0362\,Re_L^{0.8}}{Pr^{0.6}} \times$$

$$\times\left[\frac{(d_0\bar{x}+b)^{2.25} - b^{2.25}}{2.25\,d_0}\right]^{0.8} \frac{1}{d_0\bar{x}+b}. \tag{9}$$

If b = 0,

$$Re_T^{**} = \frac{0.0188\,Re_x^{0.8}}{Pr^{0.6}}$$

and, moreover,

$$St_0 = \frac{0.0338}{Re_x^{0.2}\,Pr^{0.6}}. \tag{10}$$

For

$$q_w = \text{const}$$

$$Re_T^{**} = St_0\,Re_x, \tag{11}$$

$$St_0 = \frac{0.0302}{Re_x^{0.2}\,Pr^{0.6}}. \tag{12}$$

For

$$q_w = q_0 \exp k\bar{x}$$

$$Re_T^{**} = St_0\,Re_L \frac{1}{k}\,\frac{\exp k\bar{x} - 1}{\exp k\bar{x}}, \tag{13}$$

$$St_0 = \frac{0.0306\,(k\bar{x})^{0.2}}{Re_x^{0.2} Pr^{0.6}} \left(\frac{\exp k\bar{x}}{\exp k\bar{x} - 1}\right)^{0.2}. \tag{14}$$

Relations (8), (10), (12), (14), denoted by 1, 2, 3, and 4, respectively, in Fig. 4, are in satisfactory agreement with the experimental data.

The second series of experiments was performed for $\Delta T = b - d_0\bar{x}$, $q_w = q_0 \sin \pi\bar{x}$ (Fig. 2).

These data are represented by curve 2 in Fig. 3. Clearly, the agreement with relation (6) is less satisfactory than for the first series of experiments.

The results obtained are consistent with the conclusions of [6,7], where it is shown that on a certain interval of variation of the gradient $d\Delta T/dx$ its effect on St can be neglected. According to [6,7], the relative error in determining St when the effect of the boundary conditions is neglected satisfies an inequality that can be written in the form

$$\frac{\Delta St}{St_0} \leqslant 0.1\left[\frac{Re_T^{**}}{St_0 Re_L \Delta T}\,\frac{d\Delta T}{d\bar{x}}\right]. \tag{15}$$

For the boundary conditions $q_w = q_0 \exp k\bar{x}$ and the corresponding $\Delta T \simeq c \exp kx$ we obtain, using (6), (13)–(15),

$$\frac{\Delta St}{St_0} \leqslant 0.1\,\frac{\exp k\bar{x} - 1}{\exp k\bar{x}},$$

$$\frac{\Delta St}{St_0} \leqslant 0.1,$$

or

$$\frac{Re_T^{**}}{Re_L St_0}\,\frac{d\Delta T}{\Delta T d\bar{x}}$$

since k > 0, $0 < \bar{x} < 1$. A similar result is observed for $\Delta T = b + d_0\bar{x}$. Using (6), (9), we find that

$$\frac{Re_T^{**}}{St_0 Re_L \Delta T}\,\frac{d\Delta T}{d\bar{x}} = 0.55\left[1 - \left(\frac{b}{d_0\bar{x}+b}\right)^{2.25}\right]. \tag{16}$$

Since

$$\frac{b}{d_0\bar{x}+b} \leqslant 1, \text{ we have } \frac{\Delta St}{St_0} \leqslant 0.055.$$

However, the error in determining St from (6) increases if the boundary conditions correspond to a decrease in the wall temperature or heat flux along the surface. In fact, for the case $\Delta T = b - d_0 x$, by analogy with (16), we obtain

$$\frac{Re_T^{**}}{St_0 Re_L \Delta T}\,\frac{d\Delta T}{d\bar{x}} = 0.55\left[\left(\frac{b}{b - d_0\bar{x}}\right)^{2.25} - 1\right].$$

Since

$$\left(\frac{b}{b - d_0\bar{x}}\right) \geqslant 1, \text{ we have } \frac{\Delta St}{St_0} \geqslant 0.055.$$

Thus, we may consider it established that the heat transfer law $St_0 = f(Pe_T^{**})$ is practically independent of the boundary conditions $\Delta T = f(\bar{x})$, $q_w = f(\bar{x})$, if ΔT and q_w increase along the surface.

At the same time, the effect of the boundary conditions may be appreciable if ΔT and q_w decrease downstream. Final quantitative conclusions can be reached only after additional research. It should be noted that there is some correspondence between the results obtained and the known fact that the friction law $C_{f_0} = f(Re^{**})$ is affected by the longitudinal pressure gradient, the form parameter of the thermal boundary layer

$$\frac{Re_T^{**}}{Re_L St_0}\,\frac{d\Delta T}{\Delta T d\bar{x}}$$

being analogous to that of the dynamic boundary layer

$$\frac{2\,\mathrm{Re}^{**}}{C_{f_0}\,\mathrm{Re}_L}\,\frac{1}{w_0}\,\frac{dw_0}{d\bar{x}}.$$

NOTATION

δ_T^{**} is the energy thickness; w_0, ρ_0, and T_0 are the velocity, density, and temperature at the outer edge of the boundary layer; μ_0, a, c_{p_0} are the dynamic viscosity, thermal diffusivity, and specific heat at the temperature T_0; L is the scale length; T_w is the wall temperature; q_w is the heat flux at the wall; Ψ is the relative Stanton number; Pr is the Prandtl number; St_0 is the Stanton number under standard conditions; C_{f_0} is the coefficient of friction; Re** is the Reynolds number based on the momentum thickness; d_0, b, k, and q_0 are constants.

REFERENCES

1. S. V. Patankar and D. B. Spalding, TWF (TN), 7 October 1965.
2. E. R. G. Eckert, P. I. Hartnett, and R. Birkebak, J. Aeronaut. Sci., 24, 549, 1957.
3. W. B. Hall and P. H. Price, International Developments in Heat Transfer, Pt. III, Colorado, USA, 1961.
5. I. Aoki, T. Takahaski, and H. Shimazu, Bulletin of the Tokyo Institute of Technology, no. 67, 1964.
6. V. M. Ievlev, DAN SSSR, 86, no. 6, 1952.
7. V. M. Ievlev, DAN SSSR, 87, no. 1, 1952.
8. G. S. Ambrok, ZhTF, 27, no. 9, 1957.
9. S. I. Kosterin, A. I. Leont'ev, and V. K. Fedorov, Teploenergetika, no. 7, 1961.
10. S. S. Kutateladze and A. I. Leont'ev, Turbulent Boundary Layer of a Compressible Gas [in Russian], Izd. SO AN SSSR, 1962.
11. B. S. Petukhov, A. A. Detlaf, and V. V. Kirillov, ZhTF, XXIV, no. 10, 1954.

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